5.4 Midsegment Theorem

Goals • Identify the midsegments of a triangle.
• Use properties of midsegments of a triangle.

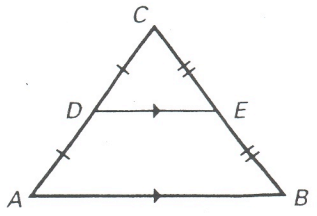
VOCABULARY

Midsegment of a triangle

THEOREM 5.9: MIDSEGMENT THEOREM

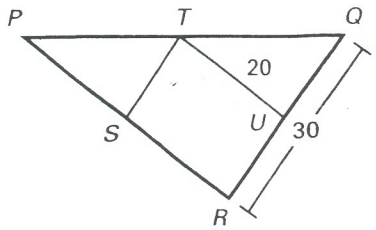
The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long.

$\overline{DE} \parallel$ ____ and $DE = \frac{1}{2}$ ____



Example 2 *Using the Midsegment Theorem*

$\overline{ST}$ and $\overline{TU}$ are midsegments of $\triangle PQR$. Find PR and ST.

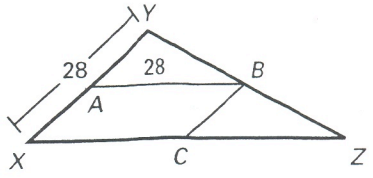


Solution

$PR = 2(\_\_\_\_) = 2(\_\_\_\_) = \_\_\_\_$

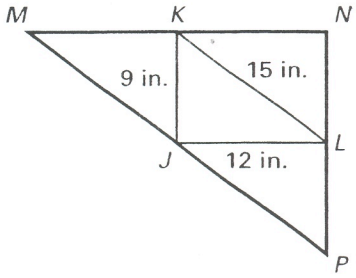
$ST = \frac{1}{2}(\_\_\_\_) = \frac{1}{2}(\_\_\_\_) = \_\_\_\_$

2. $\overline{AB}$ and $\overline{BC}$ are midsegments of $\triangle XYZ$. Find XZ and BC.



Example 4 *Perimeter of a Triangle*

$\overline{JK}$, $\overline{KL}$, and $\overline{JL}$ are midsegments of $\triangle MNP$. How does the perimeter of $\triangle MNP$ compare to the perimeter of $\triangle JKL$?



Solution

The lengths of the sides of $\triangle MNP$ are twice the lengths of the midsegments.

$MP = 2(\_\_\_\_) = 2(\_\_\_\_) = \_\_\_\_$ in.

$MN = 2(\_\_\_\_) = 2(\_\_\_\_) = \_\_\_\_$ in.

$NP = 2(\_\_\_\_) = 2(\_\_\_\_) = \_\_\_\_$ in.

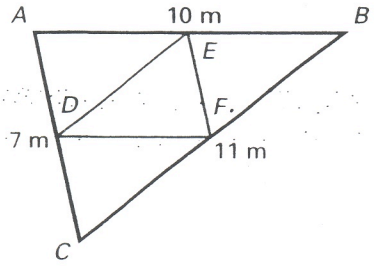
The perimeter of $\triangle MNP$ is $\_\_\_\_ + \_\_\_\_ + \_\_\_\_ = \_\_\_\_$ inches.

The perimeter of $\triangle JKL$ is $\_\_\_\_ + \_\_\_\_ + \_\_\_\_ = \_\_\_\_$ inches.

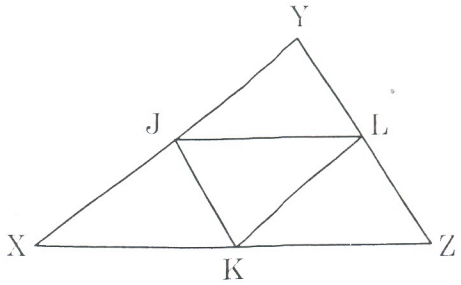
Answer The perimeter of $\triangle MNP$ is _____ the perimeter of $\triangle JKL$.

Checkpoint **Complete the following exercise.**

4. $\overline{DE}$, $\overline{EF}$, and $\overline{DF}$ are midsegments of $\triangle ABC$. Find the perimeters of $\triangle ABC$ and $\triangle DEF$. How does the perimeter of $\triangle DEF$ compare to the perimeter of $\triangle ABC$?



In ΔXYZ, $\overline{XJ} \cong \overline{JY}$, $\overline{YL} \cong \overline{LZ}$, and $\overline{XK} \cong \overline{KZ}$.



17) $\overline{JK} \parallel$ _____

18) $\overline{XY} \parallel$ _____

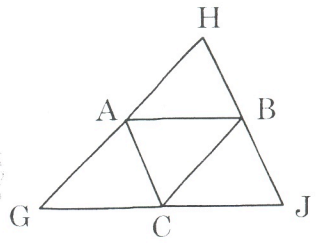
19) $\overline{JL} \parallel$ _____

20) $\overline{JL} \cong$ _____ $\cong$ _____

21) $\overline{YJ} \cong$ _____ $\cong$ _____

22) $\overline{JK} \cong$ _____ $\cong$ _____

Use ΔGHJ, where A, B, and C are midpoints of the sides.



23) If $AB = 3x + 8$ and $GJ = 2x + 24$, what is AB?

24) If $AC = 3y - 5$ and $HJ = 4y + 2$, what is HB?

25) If $GH = 7z - 1$ and $BC = 4z - 3$, what is GH?

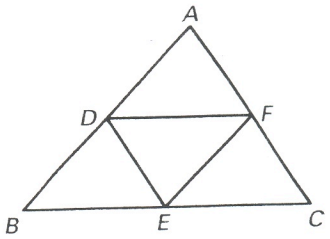
LESSON 5.4

NAME _____ DATE _____

Practice A

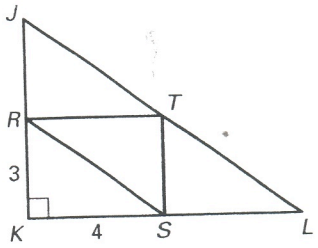
For use with pages 287–293

Use the diagram of $\triangle ABC$ where *D*, *E*, and *F* are the midpoints of the sides.



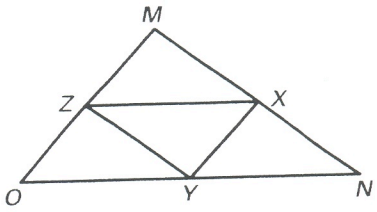
1. $\overline{DE} \parallel$?
2. $\overline{FE} \parallel$?
3. If $AB = 14$, then $EF =$?.
4. If $BE = 8$, then $DF =$?.
5. If $DE = 6$, then $AC =$?.

Use the diagram of $\triangle JKL$ where *R*, *S*, and *T* are the midpoints of the sides, *RK* = 3, *KS* = 4, and $\overline{JK} \perp \overline{KL}$.



6. Find the length of $\overline{RS}$.
7. Find the length of $\overline{JK}$.
8. Find the length of $\overline{RT}$.
9. Find the perimeter of $\triangle JKL$.
10. Name all of the right angles in the diagram.

Use the diagram of $\triangle MNO$ where *X*, *Y*, and *Z* are the midpoints of the sides.



11. If $YZ = 3x + 1$, and $MN = 10x - 6$ then $YZ =$?.
12. If $YX = x - 1$, and $MO = 3x - 7$ then $MO =$?.
13. If $m\angle MON = 48°$, then $m\angle MZX =$?.
14. If $m\angle MXZ = 37°$, then $m\angle MNO =$?.
15. Name a triangle that appears to be congruent to $\triangle ZOY$.

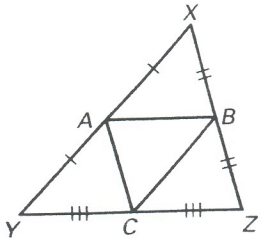
LESSON 5.4

NAME _____ DATE _____

Practice B

For use with pages 287–293

Use the diagram of $\triangle XYZ$ where *A*, *B*, and *C* are the midpoints of the sides.



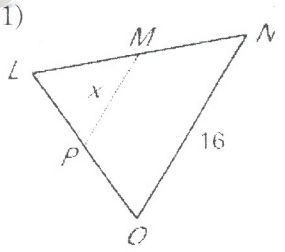
1. $\overline{AB} \parallel$?
2. $\overline{XY} \parallel$?
3. If $AC = 3$, then $XZ =$?.
4. If $YZ = 7$, then $AB =$?.
5. If $AC = 3m$, then $XZ =$?.
6. If $XY = m + 1$ and $BC = m - 3$, then $XY =$?.
7. If $AC = m - 2$ and $XZ = m + 4$, then $AC =$?.
8. If $BC = \frac{3}{4}AC$ and $XZ = 8$, then $BC =$?.

Worksheet 5.1 Midsegments

Name _____ 99

$\overline{MP}$ is a midsegment of ΔLNO. Find the value of x.

1)



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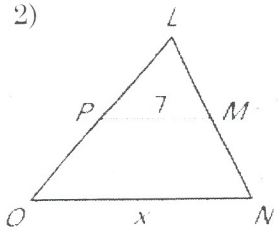

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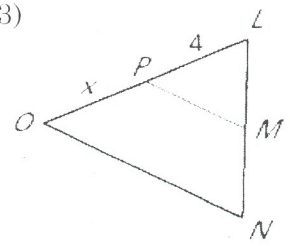

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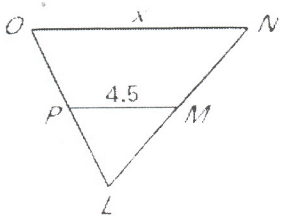


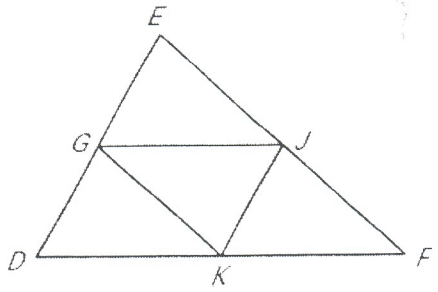
In ΔDEF, $\overline{EJ} \cong \overline{JF}$, $\overline{FK} \cong \overline{KD}$, and $\overline{DG} \cong \overline{GE}$. Complete the statements

5) $\overline{GJ} \parallel$ _____

6) $\overline{EJ} \cong$ _____ $\cong$ _____

7) $\overline{DE} \parallel$ _____

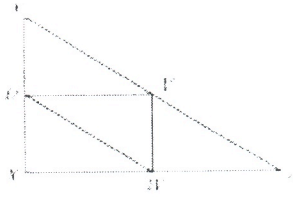
8) $\overline{GJ} \cong$ _____ $\cong$ _____



Use the diagram of ΔXYZ where U, V, and W are midpoints of the sides.

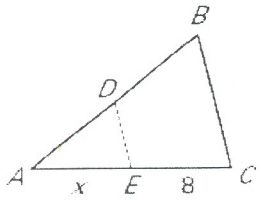
9) If $UW = 4x - 1$ and $YZ = 5x + 4$, what is UW?

10) Find YV.



$\overline{DE}$ is a midsegment of ΔABC. Find the value of x.

11)



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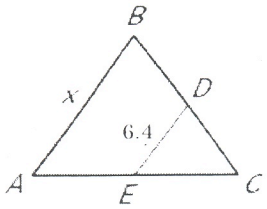
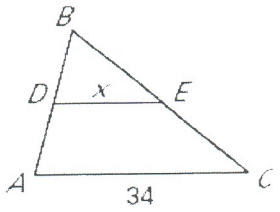


13)

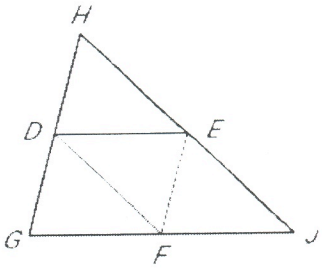


Use ΔABC, where D, E, and F are midpoints of the sides.

14) If $DE = 4x + 5$ and $GJ = 3x + 25$, what is DE?

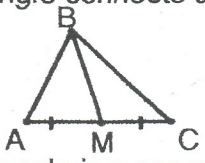
15) If $EF = 2x + 7$ and $GH = 5x - 1$, what is EF?

16) If $HJ = 8x - 2$ and $DF = 2x + 11$, what is HJ?

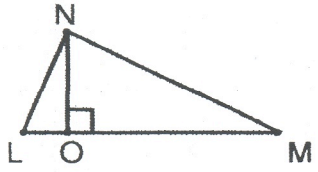


Special Segments in Triangles

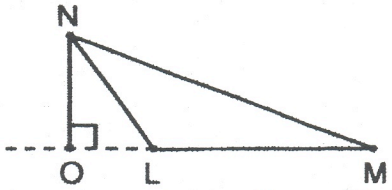
A median of a triangle connects a vertex to the midpoint of the opposite side.



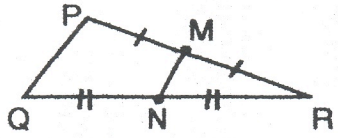
An altitude of a triangle is a segment drawn from a vertex perpendicular to the opposite side (or an extension of it).



or



A midline of a triangle connects two midpoints of two sides and is parallel to the third side. Its length is also half the length of the third side.

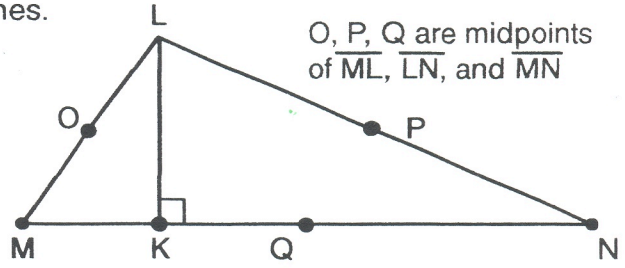


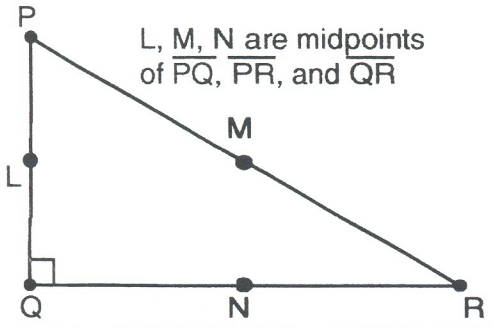
$\overline{MN} \parallel \overline{PQ}$

$MN = \frac{1}{2}PQ$

Refer to the diagrams. Find the answers in the decoder to reveal the name of the last major league baseball field to install lights for night games.

1. $\overline{LK}$ is an _____.
2. $\overline{MP}$ is a _____.
3. The median from L is segment _____.
4. $\overline{PQ}$ is a _____.
5. The median from N is segment _____.
6. If OP = 10 then segment _____ = 20.





7. The altitude from P is segment _____.
8. If PL = 10, then the length of LQ is _____.
9. If PQ = 30, then the length of MN is _____.
10. If PM = 20, then the length of LN is _____.
11. Segment LN is half the length of segment _____.
12. ΔPQR is a _____ triangle.
13. If LM = 15, then the length of QR is _____.
14. If the perimeter of ΔPQR = 50 units, the perimeter of ΔLMN would be _____ units.

10	15	20	25	30	median	midline	altitude	PQ	PR	MN	LQ	ON	right	isosceles
A	C	D	E	F	G	H	I	L	N	O	R	W	Y	Z

5	3	1	2	7	14	12	13	1	14	7	10	

1	11	9	4	1	9	8	2	6